# USING GENERATIVE ROUTINES TO SUPPORT LEARNING OF AMBITIOUS MATHEMATICS TEACHING

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In this paper, we integrate a set of theoretical considerations that together serve as a model for investigating how high-leverage practices could be generative of teacher learning. We use the context of rehearsals to investigate how the use of a specified question sequence aimed at eliciting student mathematical thinking can afford opportunities for novices and instructors to consider goals of ambitious mathematics teaching. In our results, we provide thematic categories for the problems that arose as novices used the sequence of questions, and demonstrate how they afforded the teacher educator opportunities to connect novices' work to goals of ambitious mathematics teaching. In particular, we highlight how these opportunities arose in the midst of modifying to the question sequence and investigating the consequences of its enactment.

Keywords: Teacher Education-Preservice, Rehearsals, Classroom Discourse, Instructional Activities and Practices

Much of the work of teaching is non-routine, requiring a capacity to improvise in the midst of contingent interactions, marshalling knowledge and skill in the service of professional goals (Grossman, Compton, et al., 2009). Mathematics teachers, for example, must make judgments about how to respond to students individually and in groups, drawing on specialized knowledge of both mathematics and student thinking to further instructional objectives. All the while, they must treat all students as sensemakers and provide them with access to cognitively demanding tasks. For mathematics teacher educators, this problem of complexity is associated with another problem, one of enactment. Novice teachers must learn not only to analyze teaching but also to enact it. Some current approaches to teacher education employ "pedagogies of enactment" to engage novices directly in the interactive work of teaching (Grossman & McDonald, 2008). Within these pedagogies, teacher educators are organizing teacher learning around a set of core teaching practices derived from research on student learning and professional standards. These practices include, for example, eliciting and responding to student reasoning, representing student thinking, orienting students to each other's ideas, and attending to students' errors.

To help novice teachers learn to implement these practices, there is also increasing interest in developing enactment tools, such as talk moves or specific activity frameworks. Tools translate abstract conceptual tasks into more concrete steps and objectives (Wertsch, 1998), supporting the user in implementing particular practices toward a goal. There is concern, however, that a focus on enactment tools may reduce teaching to a set of techniques, without attention to important purposes and commitments that guide teachers' practice (Kennedy, 2015). In this study, we conceptualize the idea of "generative routines" as tools that support beginners to enact core teaching practices while simultaneously learning to use goals and professional commitments to guide decision-making. We ground the idea of generative routines in Hatano and Inagaki's (1986) notion of *adaptive expertise*, where they distinguish between routine and adaptive experts and argue that the latter are those for whom performance of procedural skills is enhanced by an understanding of their purposes. Adaptive performance, they argue, requires developing both efficiency in routines and the professional knowledge and judgment to be able to innovate and adapt to new situations. Research on the development of expertise suggests that this balance is achieved through deliberate practice (Ericsson,

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Krampe, & Tesch-Römer, 1993), which allows the developing practitioner to gradually refine specific aspects of performance through cycles of repetition with feedback.

Our study examines how generative routines can mediate novice teacher learning of ambitious mathematics teaching. We focus on a particular routine, a well-specified question sequence designed to support novices in the beginning work of eliciting and responding to multiple student strategies in mathematics: What did you see (or get)?  $\rightarrow$  Did anyone see (or get) anything different?  $\rightarrow$  How did you see it (or figure it out)?  $\rightarrow$ Did anyone see it (or figure it out) in a different way? As initial prompts, these questions serve a technical purpose by providing the novice with a set of moves to elicit a range of ideas that represent student thinking and their different levels of understanding. As a result, the novice utilizing this tool is confronted with a suite of demands associated with responding to students' contributions that arise in the spaces in between consecutive questions.

Our overarching research question is *How can an enactment tool be generative for mathematics teacher learning?* More specifically, we address this question by investigating (1) the problems of practice that arise for novices in the context of using the question sequence, and (2) how these problems afford opportunities for novices and instructors to connect the sequence to goals of ambitious mathematics teaching. We focus on problems of practice because research suggests that they open spaces that are generative of teacher learning (Horn & Little, 2010).

### **Theoretical Framework**

We conceptualize learning to teach as increased participation in a community of practice where people coordinate their efforts to accomplish culturally-valued activities using tools that mediate goal-directed actions and shared cultural understandings. This sociocultural perspective on learning posits that there is circularity between tool use and the learning it is meant to facilitate (Sfard & McClain, 2002). Cultural tools mediate a learner's participation in a practice while being themselves products of this process. A key aspect of this mediation process is the way tools direct participation toward various goals around which activities are organized. Wertsch (1998) theorizes two complementary ways in which tools mediate activity. First, a tool mediates action by translating what may stand as an abstract conceptual problem for a beginner into a series of concrete operations at which one can become proficient. Thus, learning to use a tool entails developing technical skills. Wertsch also argues that a tool can support enactment through the affordances (and constraints) it contributes to the development of goal-directed activity. In the case of practice routines, affordances arise when the use of a routine towards particular ends opens up "problem spaces," problem solving situations in which the user can work through her understandings of particular concepts (Salomon & Perkins, 1998).

#### **Context and Methods**

The context of this study is a mathematics methods course taught by two teacher educators, designed around a summer learning institute (SLI) that provides four weeks of daily remedial instruction in mathematics and language arts for approximately 140 rising third graders of variable mathematics skills. The institute serves as a field setting for twenty-five novice teachers. To prepare for, and subsequently learn from, their work with children, novice teachers participated in daily Cycles of Enactment and Investigation of instructional activities that are common to the elementary mathematics curriculum, designed for novices to work on principled instructional practices and mathematical knowledge in integrated ways (Lampert et al., 2013).

Each cycle begins with the novice teachers observing and analyzing an enactment of an instructional activity (IA) in a classroom context, either live or on video. Following the observation and analysis of the IA, novices next prepare to teach it to the SLI students, rehearsing it first publicly in front of their peers and the teacher educator who participate as students, exhibiting understanding

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of how children think about disciplinary ideas. The teacher educator acts as coach, enabling both the rehearsing novice and others in the group to study a range of actions a teacher might take in response to particular student performances. All novice teachers then enact the IA with students in elementary school classrooms, video-recording themselves and writing analytic essays on their own performance. Continuing the cycle, the teacher educator guides a collective analysis using records of the enactments.

## A Question Sequence as a Generative Routine

In ambitious teaching, student ideas and contributions are the essence of mathematics discourse; thus, a teacher's work to elicit multiple student conjectures is particularly crucial. However, this practice is often at odds with many beginning teachers' instincts to seek correct answers. For this reason, we specified a sequence of four questions to be used routinely across different IAs to provide a beginning structure for some of the initial elicitation work novices must do to facilitate collective problem solving within instructional activities.

The first two questions, "What did you get?" and "Did anyone get a different answer?" (or variations of these questions), enable novice teachers to start gathering a set of possible solutions from multiple students while responding in a non-evaluative manner that positions different answers as conjectures for the group to evaluate. Once a representative set of conjectures has been elicited, the second pair of questions, "How did you figure that out?" and "Did anyone figure it out a different way?" provide novices with initial prompts to begin to elicit students' reasoning about these strategies. These initial prompts serve a technical purpose by providing the novice with a set of moves to elicit a representative range of student thinking. As a result, the novice utilizing this tool is confronted with a suite of demands associated with responding to students' contributions that arise in between the consecutive questions. Demands include pressing students to articulate their reasoning, establishing productive exchanges among students around key mathematical ideas, and representing different contributions clearly for collective consideration (Staples, 2007). These demands constitute a rich problem space associated with responding purposefully to student contributions that can be worked on collectively in rehearsals. In this way using the sequence as a tool creates affordances for novices to experiment with adapting to student performances.

## **Data Sources and Analytic Procedures**

We analyzed 19 video-recorded rehearsals, representing the rehearsals facilitated by one of the teacher educators during the second and fourth weeks of the SLI. These two weeks of rehearsal videos were selected due to the prominence of the focal question sequence in the structure of the IAs being rehearsed.

To analyze the rehearsal videos, we used Studiocode©, a software package that connects analytic codes directly to segments of video. We identified all rehearsal segments in which the question sequence (QS) in its entirety was being rehearsed, hereafter referred to as QS segments, and then narrowed in on portions of these segments where there were pauses in the simulation for exchanges between the teacher educator (TE) and novice teachers (NTs). We refer to these sub-segments as TE/NT exchanges. There were 72 TE/NT exchanges within QS segments in our data set. Guided by our theoretical framework, we characterized the problems of practice that were discussed during these exchanges as a direct result of using the question sequence. To characterize this set of problems, we began with the two broad conceptual categories theorized by Wertsch (1998) and noted earlier: problems related to technical aspects of using the question sequence; and problems afforded through its use, i.e. problems arising as a consequence of asking questions in the sequence and eliciting student responses. Problems associated with aspects of practice unrelated to the question sequence were categorized as "other" and excluded from subsequent analyses. We then followed a

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process of thematic analysis (Boyatzis, 1998) to identify sub-categories, developing short descriptions of the problems worked on in each exchange and labeling them for themes. We then discussed our emergent thematic categories to refine a final set of inductive sub-categories for each of the two broad categories.

Inside both broad categories of problems, we then used open coding to characterize how participants drew on goals and professional commitments of ambitious teaching in addressing problems of practice. We developed analytic vignettes (Erickson, 1986) for a representative set of the TE/NT exchanges in order to characterize how a teacher educator can leverage work with an enactment tool, like the question sequence, to create opportunities to explicitly connect novice teachers' work on eliciting and responding to a set of professional commitments like treating all students as sensemakers and providing equitable access to content (see Ghousseini, Beasley, & Lord (2015) for more details).

#### Results

Our analysis demonstrates how the use of the question sequence as an enactment tool, both in its technical aspects and through the problem spaces afforded by its use, brought forward a number of problems of practice that the novice teachers and teacher educator then collaboratively addressed in the rehearsal context. In managing these problems, participants engaged in a form of inquiry during which the teacher educator guided novice teacher participation in considering questions and solutions related to the use of the tool and its consequences. In the process, the teacher educator had repeated opportunities to connect judgments about adapting the question sequence to commitments of ambitious teaching, like providing students equitable access to learning and treating them as sensemakers. The problems of practice related to the technical use of the QS emerged when novices considered both adaptations to the *wording* of specific questions within the sequence and to the *order* of these questions. The problems related to the affordances of using the QS emerged when the NTs had to manage unanticipated student responses. Such responses required the TE and NTs to determine how to respond to student solutions, support their collaboration, and represent their strategies. We share illustrative vignettes from each of these categories of emergent problems.

# Problems Related to Technical Aspects of the QS

How to adapt the order of the questions in the QS. During one rehearsal, novice teachers practiced the IA of Quick Images, which focuses on helping students determine the total number of items in two ten-frames that are flashed quickly. Specifically, the novices used the QS to engage students in using the five- or ten-structure of the ten-frames to determine the total quantity. A novice teacher asks, "As we go down the list of questions, if the first student [who was] asked to explain their strategy is understanding the five- and ten- structure, can we just stop at that part, or do we need to [use the other questions in the QS] to ask for different strategies?" In other words, the problem of practice that the novice teacher is considering is "If the teacher hears the correct answer to the problem and the student communicates sound reasoning about it, is it necessary to solicit different answers and strategies?" In her question, the novice mentions the instructional goal of the ten-frame activity as a way of legitimizing the problem of practice that she is bringing forward for everyone's consideration. The TE, in response, provides several reasons why it still makes sense to continue with the next question in the sequence.

TE: So you want to look for other strategies because you want to find out as much about what students are thinking as you can, because you are still kind of assessing. And it is not about "this is the only strategy that is legitimate." There are other strategies that are legitimate and valid strategies.

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NT: But if we identify a student who is understanding the five and ten structure, could we not ask him to explain to the students too?

TE: That would be one move, or you could have other students revoice that strategy and see if they understand it.

This example illustrates the way practicing the question sequence afforded opportunities to attend to novices' personal understandings of the goals of teaching. In this example, the NT seems to be operating with the assumption that teaching is about getting quickly to an explanation of the correct answer. Ambitious teaching rests on a different set of assumptions—like the importance of investigating alternative explanations and incorrect answers—which the TE can negotiate in exchanges like this one. The TE's response stresses that the question sequence serves multiple goals beyond merely identifying the correct answer, and the correct strategy. It also emphasizes some aspects of the commitments of ambitious teaching: the importance of knowing the students as learners (finding out what they know and making instructional decisions accordingly) and treating students as sensemakers (legitimating different ways of reasoning about mathematics). Her response also underscores the importance of allowing students to collaboratively judge what is to be taken as shared. For instance, she suggests that even when a student proposes a correct strategy, the teacher should orient other students to his thinking and give them the space to make sense of it.

## Problems Related to the Affordance Aspect of the QS

Representing students' ideas as a result of using the OS. In this example, the novice teacher is faced with a situation that, from an ambitious teaching perspective, demands that she respond to a student contribution in a way that makes their thinking visible to other students and connects it to the mathematical goals of the lesson. This situation occurred during the third rehearsal of a Quick Images activity. The rehearsing novice teacher (R-NT) has flashed a card that showed 12 dots (a full ten-frame on the left side of the card, and another ten-frame with only two dots on the right side). She asks a variation of the question "How did you figure that out?" to elicit a student's strategy for recognizing that a full ten-frame and two more dots was twelve in total: "How did you see 12?" As one student explains that she saw 12 as "the full ten-frame and two more," the R-NT attempts to represent the strategy on the card, roughly pointing with her fingers to the full ten-frame and then the two dots, while saying "10 and 2." The TE deems the R-NT's response appropriate by noting, "What you just did was a good idea, to use your finger [to represent the strategy on the card]." However, the TE points out that the manner in which the R-NT has represented the strategy on the card did not convey meaningful mathematical ideas to the students because she did not deliberately point to where "10" was on the card. One goal of this Quick Images activity is to help all students see that the ten-frame represents 10, which can be done by highlighting that the top and bottom rows each contain 5 dots when they are filled, and together the two rows add up to 10. The TE's comment indirectly underscores a key commitment of ambitious mathematics teaching—to provide equitable access to learning by visually representing student strategies for collective consideration, and to target particular mathematical concepts.

The TE then directs the R-NT to replay her response to the student strategy and practice using her finger more deliberately, tracing with her fingers where the 10 is while revoicing the student's strategy. Before she replays her response, however, the R-NT raises a concern about her own pattern of response in this kind of situation:

- R-NT: Should I ask, umm, I feel when I [represent her idea in this way] that I validate her answer by saying it.
- NT: Yeah, I was doing that yesterday in my class. I was repeating everybody's answer. I don't think that's what we're supposed to be doing.

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R-NT: Like her answer will only be valid if the teacher says it again.

TE: So what would you do instead?

Two problems of practice are identified here by the NTs: one is concerned with how a teacher's revoicing move may be unintentionally interpreted by students to be a form of validation of particular answers; the other relates to the frequency of teacher revoicing of students' answers. Herbel-Eisenmann, Drake, and Cirillo (2009), in fact, documented similar concerns on the part of their inservice middle school teachers. They argued that teachers face dilemmas in using revoicing; they worry that an unintended function of revoicing could be to shift ownership of mathematical ideas from the students to the teacher. As a result, students may stop listening to their classmates and simply wait for the teacher to repeat different ideas. These unintended consequences of revoicing operate counter to a commitment of ambitious teaching: to treat students as sensemakers by giving them ownership of their intellectual work. By voicing their concerns in this example, the NTs seem to be acting on an implicit awareness of this commitment of ambitious teaching. With her question "What would you do instead?" the TE guides the NTs' participation in considering the use of revoicing as a form of representation of student thinking. The R-NT, in response, offers that as the student is explaining a strategy, she could just represent it on the card without doing a lot of talking. The TE directs her to try it; however, as the R-NT replays her response to test it out, she mainly represents the student strategy (of adding ten and two more) by roughly pointing to the ten frames on the card. Her replay of the response opens up an opportunity to investigate its consequences. The investigation starts when the TE intervenes again.

TE: So, the reason why you have to [restate the student strategy] is because what you're doing is you're taking her strategy and making it accessible to all the students by representing it on the card.... If you were just to use your finger, it's not- it's not connecting her strategy to what's on the card. So, you kind of have to say it in a way. Does that make sense?"

What the TE underscores in her intervention is that, in this case, the R-NT's response must support students' understanding of the meaning of "ten" by helping them connect verbal and pictorial representations of it. In this way, the TE connects the work of eliciting and responding to a guiding commitment of ambitious teaching: that giving equitable access to learning requires making explicit the different mathematical ideas that are shared during the lesson explicit.

In another example also taken from a Quick Images activity, a R-NT was standing in front of a white board, flashing different cards to a group of seven students who are sitting in a semi-circle. After flashing a card representing the problem 9 + 4, and asking the first two questions in the elicitation sequence (What did you get? Did anyone get anything different?), the R-NT gets two responses, 13 and 14. In responding to the two different answers, she turns the card face up for students to check their answers. However, as she does that, she orients her body and the card in the direction of one student who had incorrectly seen 14 dots. Given that this student was sitting at one end of the semi-circle, the TE intervenes, noting that by mainly angling the card toward that student, the R-NT was limiting the access of the students sitting at the other end of the semi-circle. She explains,

- TE:Remember, this is about everybody processing. So be careful not to walk over and make it about you and this student. So if you could stay [in the center of the semi-circle] and show the face of the card so everyone can see. You can ask her to explain [while you are standing there].
- R-NT: They really wanted to—and I made this mistake yesterday—they wanted to come up and show everybody how they did it. And I was letting them, but I think that I would not let them do this anymore.

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TE:I would try to cut that out. You can just tell them "explain to me with your words," and just help them articulate their strategy.

In this instance, the novice gets in a problem space after trying to respond to the two strategies that she elicited with the first two questions of the sequence. The problem of practice that is at play pertains to managing the position of the representation so that students can have access to the mathematical ideas that the group is attempting to address. Positioning the representation in a way that allows every student to see it communicates to them that they are all accountable for judging the reasonableness of answers. Twice in her intervention, the TE reminds the R-NT about this important commitment of ambitious teaching: "Remember, this is about *everybody processing*" and "show the face of the card so *everyone can see it*." The R-NT's justification for her move underlines a problematic situation that she was trying to remedy and suggests that she was trying to attend to the goal of students' joint collaboration: "they wanted to come up and show everybody how they did it."

## **Discussion**

Our study illustrates the potential of deliberate practice with a generative routine for supporting the learning of adaptive performance. Generative routines, like the question sequence, can function as a stable procedure that can reduce some of the initial complexity of relational practice. At the same time, enacting the procedure opens up a rich problem space for novices, who must confront the contingencies of students' improvisational responses. As such, generative routines can be more than scripts or processes that scaffold performance. When novices have opportunities to navigate these problem spaces in the company of more experienced others, generative routines can mediate learning about goals of professional practice, including the commitments that enable practitioners' judgment in situations of uncertainty.

Our analysis reveals the important role played by the teacher educator in connecting the use of the tool to professional commitments of ambitious teaching. She participates in this role through various interpersonal engagements with the novice teachers, allowing them space for practicing the work of teaching while at the same time guiding their participation in it through various forms of interventions aimed at making explicit connections between particular courses of action and commitments of ambitious teaching. Our findings provide evidence of the teacher educator focusing the novices' attention on the problematic situations that arise in practice and framing considerations for possible solutions around particular commitments of ambitious teaching. Without the teacher educator in this early stage of novice teachers' experimentation with enactment tools like the question sequence, it may be difficult for them to recognize the problematic nature of situations and to translate their current understanding of the commitments of ambitious teaching into practice.

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